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Composite black holes in external fields

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Abstract

The properties of composite black holes in the background of electric or magnetic flux tubes are analyzed, both when the black holes remain in static equilibrium and when they accelerate under a net external force. To this effect, we present a number of exact solutions (generalizing the Melvin, C and Ernst solutions) describing these configurations in a theory that admits composite black holes with an arbitrary number of constituents. The compositeness property is argued to be independent of supersymmetry. Even if, in general, the shape of the horizon is distorted by the fields, the dependence of the extreme black hole area on the charges is shown to remain unchanged by either the external fields or the acceleration. We also discuss pair creation of composite black holes. In particular, we extend a previous analysis of pair creation of massless holes. Finally, we give the generalization of our solutions to include non-extreme black holes.

1 Introduction

The recently successful microscopic interpretation of the Bekenstein-Hawking entropy within string theory [1] has required, among other things, a deeper understanding of the black hole solutions of the low energy field equations of the theory. In particular, an important role has been played by a class of remarkably simple extremal black hole solutions with charges belonging to different gauge fields [2, 3, 4, 5]. The way in which these charges enter in the solutions, namely, through products of harmonic functions, indicates that each of the gauge fields acts independently of the others. This feature, regardless of the different stringy origin of each of the gauge fields, leads to a picture in which the extremal black hole can be viewed as a composite of three (in five dimensions) or four (in four dimensions) 'constituent' black holes, each of the latter possesing charge under only one of the gauge fields.

The simplest generalization of Einstein-Maxwell theory that is relevant to superstring and supergravity theories is obtained by introducing a scalar (dilaton) field ϕ whose coupling to the Maxwell field is measured by some constant a. The action takes the form

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big\{R - 2(\partial\phi)^2 - e^{-2a\phi}F^2\Big\}. \tag{1.1}$$

The fields F and ϕ are in general linear combinations of the large variety of gauge fields and scalars of the underlying theories. Black hole solutions of the equations of motion of these theories were found in [6]. For $a > 0$, the event horizon of the black holes in the extremal limit shrinks to zero size and becomes singular. However, it was found in [7] that if one considers a theory with two different gauge fields (and $a = 1$), then the extremal horizon is regular as long as the charges under each $U(1)$ are both non vanishing. These $U(1)^2$ theories of [7] can in turn be embedded in the following $U(1)^4$ theory, which arises as a truncation of low energy heterotic string theory compactified on a six-torus [4]

$$\begin{aligned} I &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big\{R - \frac{1}{2}\left[(\partial\eta)^2 + (\partial\sigma)^2 + (\partial\rho)^2\right] \\ &- \frac{e^{-\eta}}{4}\left[e^{-\sigma-\rho}F_{(1)}^2 + e^{-\sigma+\rho}F_{(2)}^2 + e^{\sigma+\rho}F_{(3)}^2 + e^{\sigma-\rho}F_{(4)}^2\right]\Big\}. \end{aligned} \tag{1.2}$$

The composite black hole solutions (to be reviewed below) admit also different embeddings into string theory other than toroidally compactified heterotic strings, as well as in eleven-dimensional supergravity (see, e.g., [5] and refs. therein). This action, and its extreme black hole solutions, has been used to argue that the extreme black holes of (1.1) with $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ consist of, respectively, 1, 2, 3, or 4 particle states bound at threshold [4, 8]. It should be noted that these particular values of a are the only ones for which the black holes can be consistently embedded in maximal supergravity theories.

We will also propose another class of actions, which generalize the $U(1)^4$ theory above, and which will be used to construct composites of an arbitrary number of extreme black holes. Specifically, the theories contain n gauge fields and $n - 1$ independent scalars with

action

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big\{R - \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\partial\sigma_i - \partial\sigma_j)^2 - \frac{1}{n}\sum_{i=1}^{n} e^{-\sigma_i}F_{(i)}^2\Big\}, \tag{1.3}$$

and the scalars satisfying

$$\sum_{i=1}^{n}\sigma_i = 0. \tag{1.4}$$

This theory contains, modulo electric-magnetic duality rotations of the gauge fields, the $U(1)^2$ theories of [7] and the $U(1)^4$ theory above [1]. The extremal black hole solutions of the single-scalar-Maxwell theory with any rational value of a^2 will be seen to arise as composites of several of these black holes. In all cases the binding energy will be zero.

The action (1.3) does not, in general, arise from any truncation of compactified string theory, but it might instead appear from some truncated form of extended supergravity coupled to matter supermultiplets. Though we have not found such embedding, the extremal black hole solutions of (1.3) present a number of features in common with supersymmetric black holes. These include saturation of BPS-like bounds, multi-center solutions and attractors in moduli space. The solutions are found to admit simple forms even for an arbitrary number of gauge fields, and this will allow a study in wide generality of the properties of composite black holes.

The main goal of this paper will be to probe the behavior of composite black holes in external fields. To do so, we have to analyze how to describe the geometry of these black holes in an external field. In a theory containing gravity a uniform electric or magnetic field does not exist: the energy stored in the field bends the spacetime, and the most we can aspire to is an axisymmetric configuration describing a flux tube with field strength approximately uniform on distances to the axis smaller than the magnetic length $1/B$. This is the Melvin solution of General Relativity [10]. As will be shown below, the theories (1.2) and (1.3) also admit Melvin flux tube solutions, with the fields taking the form of composites.

What can we expect to happen to a composite black hole when placed in a background field? First of all, we have each of the $U(1)$ charges q_i coupled to a background flux tube of strength B_i, and thus subject to a force given (roughly) by q_iB_i.[2] We can therefore achieve an equilibrium configuration for the black hole if the forces are balanced so that their total cancels out. One may wonder whether the composite should not break apart, given the zero binding energy between constituents. This question turns out to be not easy to ascertain. In the solutions of black holes in external fields that we will present, the constituents remain bound almost by construction. The problem is that the supersymmetry of the extreme solutions without external fields that guarantees their stability and the cancellation of forces

[1]The theory (1.3) is closely related to the multi-scalar theories considered in [9]. However, the constraint (1.4) that we are imposing implies that we consider, in general, their matrix $M_{\alpha\beta}$ to be singular. The only such case considered in [9] in 4 dimensions is the $U(1)^4$ theory, embeddable in a maximal supergravity theory.

[2]The charges and fields can be either electric or magnetic, but we will indistinctly denote them by q_i and B_i.

between constituents, is completely broken by the introduction of the background field, which does not possess any Killing spinors. We envisage two possibilities:

a) The solution describing the composite black hole in equilibrium under the pull of different $U(1)$ fields could be classically unstable, evolving perhaps towards a configuration where the composite is broken. The analysis of this possibility, that involves studying the spectrum of fluctuations of the solution, is certainly extremely complicated due to the low degree of symmetry of the solution. Even more, the stability analysis could change if we embedded the black hole in a higher dimensional theory. However, the classical black hole theorems forbid the possibility of classical splitting of the regular extreme black hole (in our case, with n non-vanishing charges). Since partially charged black holes have singular horizons of zero area, such a process could be seen as violating the second law of black hole mechanics/thermodynamics. Hence the regular extreme black hole could be expected to be classically stable. This reasoning, though, does not apply to the singular, partially charged extreme black holes. There also remains the possibility of quantum splitting, though it is more unlikely.

b) Since supersymmetry is broken, the balance of forces between constituents is not expected to hold anymore, and they could attract each other and compensate the pull from the external fields. Again, this is difficult to check, since some of the properties of the black holes, like their mass and scalar charges, are not well defined in the presence of an external field, and, even worse, we can not even define the mass and scalar charge of each individual constituent.

Even if, at present, we cannot solve this stability issue, there are other questions to be analyzed in these black hole + external field configurations. As we will show, the shape of the black hole happens to be distorted by the external field. From a classical, macroscopic perspective it is not clear whether its area should remain unchanged. But if the area is to be identified with an entropy admitting a microphysical interpretation, then we do not expect the number of microstates of the black hole to be altered by the introduction of the field. This question can be unequivocally addressed for extremal black holes since their entropy can be expressed (in contrast to that of non-extremal black holes) as a function of the conserved $U(1)$ charges alone. Consistency with a microstate interpretation would require that this function is not altered by the introduction of the field. We should mention that the absence of supersymmetry means that we can not reliably extrapolate to the weak coupling regime where a description in terms of strings and branes living in flat space could be possible (for the cases with a stringy interpretation). However, recent experience indicates that many quantities not clearly protected by supersymmetry can be extrapolated at least in near-supersymmetric configurations.

We have been mentioning only the case where the external forces acting upon the black hole exactly cancel out and the black hole remains static. But we could also have a situation where a net force accelerates the black hole to infinity. It is remarkable that the same property that allows the existence of composite solutions for static extremal black holes and Melvin fields, also makes it possible to describe composite black holes in accelerated

motion [11]. Using these solutions we will test further the invariance of the area (precisely, its dependence on the $U(1)$ charges) under accelerated motions. As an outcome, we will be able to construct instantons mediating the pair creation of composite black holes.

Another interesting feature of the theories considered above is that they admit a large number of massless extremal solutions. The essential point is that, due to the existence of several kinds of gauge and scalar charges, the BPS bound can be satisfied even when the mass is set to zero. In the bound state picture, zero mass is achieved by binding positive and negative mass constituents (the latter, of course, not being allowed to exist in isolation) [12]. This interpretation is often helpful, even though the existence of the massless solutions is independent of it and the quantum stability of such a bound state might be problematic.

A point of concern with these *classical* massless solutions of low energy string theory to qualify as solitons is the fact that they are nakedly singular. Within the context of black hole solutions, naked singularities usually arise when a extremality condition, e.g., $M \geq |Q|$ for Reissner-Nordstrom solutions, is violated. Now, this bound can be thought of as a Bogomolnyi bound resulting from partial breaking of extended supersymmetry, and therefore one could try to exclude nakedly singular Reissner-Nordstrom solutions on supersymmetry grounds alone [7] (in this simple form, though, this argument does not eliminate singular supersymmetric Kerr-Newman solutions [13]). The classical singular massless solutions we are discussing do not arise as limits of non-extreme black holes, but still saturate the supersymmetric BPS bound (in the cases where a supersymmetric embedding is known). Thus the argument could, perhaps, be advocated that if solutions are to be censored on the basis of supersymmetry arguments, then the extremal massless holes should be admitted in the theory.

Nevertheless, one could wonder whether a black hole with Compton wavelength ($\sim 1/M$) much larger than its Schwarzschild radius ($\sim M$) can possibly admit a semiclassical description, so the consistency of including massless solutions still has to be further studied. In previous work we have found that if classical massless holes were allowed in the theory, then Minkowski space would, apparently, be non-perturbatively unstable against their production in pairs [11]. Furthermore, the instanton mediating the tunneling process was found to have negative action, thus implying an exponentially enhanced pair production rate. We will extend this result to massless holes in the $U(1)^n$ theories. One would be tempted to conclude that either some mechanism exists that forbids the pair production of massless holes, [3] or, otherwise, the classical massless solutions should be regarded as physically unacceptable.

The paper is organized as follows. We start by reviewing in Sec. 2 the solutions of four-constituent static extreme black holes, and then introduce the more general n-constituent solutions. In Sec. 3 we develop the techniques needed to generate solutions with Melvin field backgrounds and analyze the properties of static black holes in the field of a composite flux tube. More general, accelerating solutions of C- and Ernst type for composite black

[3] The question would remain, then, of how they can be created at all, since they neither seem to appear as the result of collapse or evaporation processes.

holes, are obtained in Sec. 4, where we also construct the instantons mediating composite black hole pair creation. In Sec. 5 we allow for negative mass constituents in order to find massless hole solutions, and we describe the instability of flat Minkowski space induced by these objects. In Sec. 6 we present the generalization to include non-extreme black holes, both static and in accelerated motion. Sec. 7 contains some final remarks on this work.

A word on normalization conventions. The normalization of the gauge fields $F_{(i)}$ in the $U(1)^n$ action has been found to conveniently simplify the formulas below, but it differs from the 'canonical' one by the prefactor $1/n$. The charge and field parameters in our solutions $\hat{q}_i$, $\widehat{B}_i$ are chosen so that, for instance, the field of a monopole is $F_{(i)} = \hat{q}_i \sin\theta \, d\theta \wedge d\varphi$, and a uniform magnetic field has $F_{(i)}^2 = 2\widehat{B}_i^2$. This has the odd effect that the force exerted on a charge $\hat{q}$ by a field $\widehat{B}$ is $\hat{q}\widehat{B}/n$. The usual form appears when considering the 'canonical' charges $\mathcal{Q}_i$ and fields $\mathcal{B}_i$, obtained as

$$\mathcal{Q}_i = \frac{\hat{q}_i}{\sqrt{n}}, \qquad \mathcal{B}_i = \frac{\widehat{B}_i}{\sqrt{n}}. \tag{1.5}$$

2 Static composite black holes

The extreme *electrically* charged black hole solutions of the Einstein-Maxwell-scalar theory (1.1) are [6]

$$\begin{aligned} ds^2 &= -\left(1+\frac{\beta}{r}\right)^{-2/(1+a^2)} dt^2 + \left(1+\frac{\beta}{r}\right)^{2/(1+a^2)} \delta_{mn} dx^m dx^n, \\ e^{-2a\phi} &= e^{-2a\phi_0} \left(1+\frac{\beta}{r}\right)^{2a^2/(1+a^2)}, \qquad A_t = -\frac{e^{a\phi_0}}{\sqrt{1+a^2}} \left(1+\frac{\beta}{r}\right)^{-1}, \end{aligned} \tag{2.1}$$

$r^2 = x_m x^m$, and the mass M and electric charge $e^{a\phi_0}Q$ given by

$$M = \frac{\beta}{1+a^2}, \qquad Q = \frac{\beta}{\sqrt{1+a^2}}. \tag{2.2}$$

The asymptotic value of the scalar, ϕ_0, is a free parameter. In some cases we will fix it so as to match an appropriate background at infinity. We can also define a scalar charge, Σ, by $\phi = \phi_0 + \Sigma/r + O(r^{-2})$, so that $\Sigma = -a\beta/(1+a^2)$, and the solution satisfies the 'antigravity' extremality condition

$$M^2 + \Sigma^2 = Q^2, \tag{2.3}$$

that expresses cancellation of gravitational and scalar attraction against electromagnetic repulsion. This allows multi-centered black hole solutions to exist. The *magnetically* charged solutions can be obtained by leaving the canonical metric invariant and transforming $F \to e^{-2a\phi} * F$ and $\phi \to -\phi$, with $*F$ the Hodge dual of F.

The extremal black hole solutions of the theory (1.2) take the form

$$\begin{aligned}
ds^2 &= -(\Delta_1\Delta_2\Delta_3\Delta_4)^{-1}dt^2 + \Delta_1\Delta_2\Delta_3\Delta_4\delta_{mn}dx^m dx^n, \\
e^{-\eta} &= \frac{\Delta_1\Delta_3}{\Delta_2\Delta_4},\ e^{-\sigma} = \frac{\Delta_1\Delta_4}{\Delta_2\Delta_3},\ e^{-\rho} = \frac{\Delta_1\Delta_2}{\Delta_3\Delta_4}, \\
F_{(1/3)\,tj} &= \partial_j\Delta_{1/3}^{-2}, \quad \tilde{F}_{(2/4)\,tj} = \partial_j\Delta_{2/4}^{-2}, \\
\Delta_i &= \left(1+\frac{q_i}{r}\right)^{\frac{1}{2}} \qquad i=1,\ldots,4,
\end{aligned} \tag{2.4}$$

where $\tilde{F}_{2/4} = e^{-\eta\pm(-\sigma+\rho)} * F_{2/4}$ and the scalars have been set to asymptote to zero at infinity. The charges $q_1,\ q_3$ in (2.4) are of electric type, whereas $q_2,\ q_4$ are of magnetic type, but it is clear that we can apply a duality transformation to the gauge fields and reverse the role of electric-magnetic charges. If all the q_i are (strictly) positive, then $r=0$ is a regular horizon of finite size, and there is a singularity behind the horizon at $r=-\max_i\{q_i\}$. If any of the q_i vanishes, the horizon shrinks to zero size and the surface $r=0$ is a double null singularity.

This solution was first found in [2] (see also [3]) and later rediscovered in [4]. Each of the harmonic functions Δ_i^2 can be generalized so as to give multi-center configurations. The form of this solution suggests that black hole configurations can be thought of as composites of four basic building blocks, each charged under a different Maxwell field. In particular, the black hole solutions of single scalar-Maxwell theories with $a=\sqrt{3},1,1/\sqrt{3},0$ correspond to taking, respectively, equal charges for one, two, three or four constituent black holes sitting at the same point, the rest of the charges being zero (or placed at infinity). For example, if we take $q_1=q_3=\sqrt{2}Q$ and $q_2=q_4=0$, we recover the $a=1$ solution of (2.1), with fields $\eta=2\phi$, $\sigma=\rho=0$, and $F_{(1)}=F_{(3)}=\sqrt{2}F$. Each constituent can be thought of as having a mass $m_i=q_i/4$, which is its ADM mass when all other constituents are absent. Except in section 5, we will take these parameters to be non-negative. The total mass is

$$M = \frac{1}{4}\sum_{i=1}^{4} q_i = \sum_{i=1}^{4} m_i, \tag{2.5}$$

and therefore the binding energy is zero.

The property that allows such factorized form for the solutions is closely linked to yet another extremal ‘antigravity’ condition (which is interpreted as a supersymmetric BPS condition when the theory is embedded in a supergravity theory). To see this, let us note, as in [4], that for a conformastatic ansatz

$$ds^2 = -H^{-1}dt^2 + H\delta_{mn}dx^m dx^n, \tag{2.6}$$

the Ricci tensor takes the form

$$\begin{aligned}
R_{tt} &= -\frac{1}{2H^2}\sum_k \partial_k^2 \log H, \\
R_{ii} &= -\frac{1}{2}\sum_k \partial_k^2 \log H - \frac{1}{2}(\partial_i \log H)^2, \\
R_{ij} &= -\frac{1}{2}\partial_i \log H\ \partial_j \log H.
\end{aligned} \tag{2.7}$$

The nonlinear terms would generate crossed terms between the different $U(1)$ fields, e.g., $(\partial \log \Delta_1)(\partial \log \Delta_2)$. However, it is straightforward to check that, when the scalar fields are introduced, all the crossed terms terms cancel out in the differential equations of motion if the algebraic relation

$$(\log H)^2 + \eta^2 + \sigma^2 + \rho^2 = 4 \sum_{i=1}^{4} (\log \Delta_i)^2 \tag{2.8}$$

is satisfied. For $H = \prod_i \Delta_i$, and defining the scalar charges N, Σ, P for the scalar fields η, σ, ρ by $\eta = N/r + O(r^{-2})$, etc., then (2.8) implies

$$4M^2 + N^2 + \Sigma^2 + P^2 = \sum_{i=1}^{4} q_i^2. \tag{2.9}$$

This is precisely the BPS bound that expresses the cancellation of forces among constituents.

Now, given this fact that the independent action of each of the gauge fields follows from a simple algebraic relation, we would like to construct more general theories with this property, such that a wider class of dilatonic extreme black holes with coupling a could be viewed as composites. The theory defined by (1.3) does precisely this. Extreme black hole solutions of this theory with electric charge under each of the $U(1)$ fields have metric (2.6), with

$$H = \prod_{i=1}^{n} \Delta_i, \qquad \Delta_i = \left(1 + \frac{q_i}{r}\right)^{2/n}, \tag{2.10}$$

and

$$e^{-\sigma_i} = e^{-\sigma_i^0} \frac{\Delta_i^n}{H}, \qquad F_{(i)\,tj} = \pm e^{\sigma_i^0/2} \partial_j \Delta_i^{-n/2}, \tag{2.11}$$

where we have reintroduced non-vanishing scalar vevs σ_i^0, to be of later use, and have emphasized that the sign of the electric charges $\hat{q}_i$,

$$\hat{q}_i = \pm e^{\sigma_i^0/2}\, q_i, \tag{2.12}$$

is essentially arbitrary. Upon making $F_{(i)} \to e^{-\sigma_i} * F_{(i)}$ and $\sigma_i \to -\sigma_i$, we can convert the electric charges into magnetic ones. Multi-center solutions are equally possible.

The algebraic relation that allows cancellation in the field equations of crossed terms between different gauge fields is

$$(\log H)^2 + \frac{1}{n^2} \sum_{i>j} [(\sigma_i - \sigma_i^0) - (\sigma_j - \sigma_j^0)]^2 = n \sum_{i=1}^{n} (\log \Delta_i)^2, \tag{2.13}$$

which, in turn, can be seen as an extremality condition between parameters,

$$M^2 + \frac{1}{4n^2} \sum_{i<j} (\Sigma_i - \Sigma_j)^2 = \frac{1}{n} \sum_{i=1}^{n} q_i^2, \tag{2.14}$$

where $M = \frac{1}{n} \sum_{i=1}^{n} q_i$, and Σ_i is the scalar charge associated to σ_i. The latter saturate separate bounds of the form

$$\frac{\Sigma_i}{2} + q_i = M. \tag{2.15}$$

The solution (2.10) makes it possible to interpret, for every rational value of a^2, the extreme single-Maxwell-scalar black holes (2.1) as composite objects. To this effect, notice that if s out of the n possible electric charges are equal and non-zero, say, $\hat{q}_1 = \ldots = \hat{q}_s$, with the remaining $n-s$ charges being zero, then the black hole solution corresponding to dilaton coupling

$$a = \sqrt{\frac{n}{s} - 1}, \tag{2.16}$$

is reproduced. In fact, the fields are identified as

$$\begin{aligned} \sigma_1 &= \ldots = \sigma_s = 2a\phi, \qquad \sigma_{s+1} = \ldots = \sigma_n = -\frac{2}{a}\phi, \\ F_{(i)} &= \sqrt{a^2+1}\, F, \quad i = 1, \ldots, s. \end{aligned} \tag{2.17}$$

Therefore, we can view an extreme black hole of dilaton gravity with $a^2 = p/q$ as an s-hole in the $U(1)^n$ theory with $n = p + q$ and $s = q$ [4].

Notice that the 'basic blocks' (i.e., the one-constituent solutions, $s = 1$) always correspond to $a \geq 1$ (except for the trivial case $n = 1$). This provides further evidence for the conjecture that the extreme black holes with $a < 1$ should not be viewed as elementary particles [14].

The entropy associated to the black holes in the $U(1)^4$ theory is [2]

$$S = A_{bh}/4 = \pi\sqrt{q_1 q_2 q_3 q_4} = \pi\sqrt{|\hat{q}_1\hat{q}_2\hat{q}_3\hat{q}_4|}, \tag{2.18}$$

whereas for a $U(1)^n$ black hole we find

$$S = \pi \prod_{i=1}^{n} q_i^{2/n} = \pi \prod_{i=1}^{n} |\hat{q}_i|^{2/n}. \tag{2.19}$$

Since the $U(1)^n$ theory does not presumably correspond, for $n > 4$, to any truncation of compactified string theory, it is uncertain whether the expression of the entropy as a product of charges stems from a more fundamental microscopic origin.

Notice that the entropy does not depend on the asymptotic value of the scalars. Actually, this can be related to the elegant principle to compute the black hole entropy developed by Ferrara and Kallosh, which is based on the existence of attractors in moduli space [15]. By this it is meant that near the black hole horizon the solution does not really depend on the values of the moduli at infinity. Let us explain this in a bit more detail. The extreme black hole solution is completely determined once we specify $2n-1$ independent parameters: the $n-1$ independent asymptotic values of the moduli σ_i^0 and n parameters $\bar{q}_i$ which, for

[4] One could wonder what role is played in this construction by the $(n-s)$ gauge fields that are set to zero, since also an equal number of scalar fields decouple from the theory, and thus we are effectively left with a theory with s gauge fields and scalars. However, the fact that we started with n fields, instead of s, enters through the normalization of the scalar fields in (1.3), which is fixed and depends on n. This normalization is actually all there is to the coupling constant a, since in (1.1) we could redefine $a\phi \to \phi$ so that a would only appear as fixing the normalization of the kinetic term $(\partial\phi)^2$.

reasons that will be immediately apparent, we choose as $\bar{q}_i = \hat{q}_i e^{-\sigma_i^0}$. The mass, in particular, depends on all the parameters. However, near the horizon $r \to 0$ we find

$$\begin{aligned} e^{-\sigma_i^0} &\to \frac{|\bar{q}_i|^2}{\prod_{j=1}^n |\bar{q}_j|^{2/n}}, \quad F_{(i)} \to \frac{1}{\bar{q}_i} dt \wedge dr, \\ r^2 H &\to \prod_{j=1}^n |\bar{q}_j|^{2/n}, \end{aligned} \tag{2.20}$$

so to fix the solution at the horizon we need only specify the n parameters $\bar{q}_i$, with the values of σ_i at infinity being arbitrary.

The entropy in this approach is found as follows. For theories with extended supersymmetry, the mass of the extreme black hole is equal to the modulus of a central charge of the supersymmetry algebra,

$$M = |Z(\bar{q}_i, \sigma_i^0)|. \tag{2.21}$$

In the present case, although we do not know whether such a supersymmetric embedding actually exists for arbitrary values of n, we will define, according to this relation, a quantity $|Z|$ as

$$|Z| = \frac{1}{n} \sum_{i=1}^n |\bar{q}_i| e^{\sigma_i^0/2}. \tag{2.22}$$

Then, we extremize $|Z|$ as a function of the moduli $e^{\sigma_i^0}$ at fixed $\bar{q}_i$, taking into account the constraint (1.4). The values of the moduli at the fixed point thus found are precisely given by their values at the horizon, (2.20). Now, following the prescription of [15], the entropy of the black hole is given in terms of the central charge at the fixed point as

$$S = \pi |Z_{\rm fix}|^2. \tag{2.23}$$

By substituting Eqs. (2.22) and (2.20) into (2.23), and taking into account that $\prod_{j=1}^n |\bar{q}_j| = \prod_{j=1}^n |\hat{q}_j|$, we find that the entropy (2.19) is indeed reproduced.

We shall end this section by remarking that a solution of the $U(1)^n$ theory with $n = 4$ can be converted into a solution of the $U(1)^4$ truncation of heterotic string theory upon replacing

$$\begin{aligned} g_{\mu\nu} &\to g_{\mu\nu}, \\ F_{1/3} &\to F_{1/3}, \qquad F_{2/4} \to \tilde{F}_{2/4}, \\ \eta &= \frac{1}{4}(\sigma_1 - \sigma_2 + \sigma_3 - \sigma_4), \quad \sigma = \frac{1}{4}(\sigma_1 - \sigma_2 - \sigma_3 + \sigma_4), \\ \rho &= \frac{1}{4}(\sigma_1 + \sigma_2 - \sigma_3 - \sigma_4). \end{aligned} \tag{2.24}$$

In some cases, though, given its special significance for string theories and maximal supergravity we will explicitly write the results for the $U(1)^4$ theory (1.2).

3 Black holes in composite Melvin fields

We would like now to find solutions describing the composite black holes of the previous section as subject to (approximately uniform) external fields. In [20], a systematic way was presented to generate background fields in the single-scalar-Maxwell theory (with arbitrary a) starting from axisymmetric solutions. This involved a generalization to dilaton theories of the general relativistic Harrison transformation [19]. For the single scalar, $U(1)^2$ theory, the corresponding transformation was given in [21]. Here we generalize the method, first to the $U(1)^4$ theory, then to $U(1)^n$. The remarkable composite structure will be seen to appear again.

Thus, let us be given a solution of the equations of motion of the theory in the warped product form

$$ds^2 = g_{jk}dx^j dx^k + g_{\varphi\varphi}d\varphi^2, \tag{3.1}$$

with g_{jk} a $2+1$-dimensional metric, and gauge potentials

$$A_{(i)} = A_{(i)\ \varphi}d\varphi, \qquad i = 1, \ldots, n, \tag{3.2}$$

where all the fields $g_{\mu\nu}$, $A_{(i)}$ and the scalars are independent of φ. For the $U(1)^4$ theory, the potentials $A_{(2/4)}$ are meant to be potentials for the *dual* field strengths $\tilde{F}_{(2/4)}$, i.e., $F_{(2/4)\ \mu\nu} = \frac{1}{2}e^{\eta\pm(\sigma-\rho)}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha A^\beta_{(2/4)}$.

For ease of comparison, we quote the transformation found in [20] for the single-Maxwell-scalar theory:

$$\begin{aligned}
g'_{jk} &= \lambda^{2/(1+a^2)}g_{jk}, \qquad g'_{\varphi\varphi} = \frac{1}{\lambda^{2/(1+a^2)}}g_{\varphi\varphi}, \\
e^{-2a\phi'} &= \lambda^{2a^2/(1+a^2)}\ e^{-2a\phi}, \\
A'_\varphi &= \frac{2}{(1+a^2)B\lambda}\left(1+\frac{1+a^2}{2}BA_\varphi\right), \\
\lambda &= \left(1+\frac{1+a^2}{2}BA_\varphi\right)^2 + \frac{1+a^2}{4}B^2 g_{\varphi\varphi}e^{2a\phi}.
\end{aligned} \tag{3.3}$$

When applied to a solution of the form (3.1), (3.2), these transformations leave the action (1.1) invariant, thus yielding a new solution. In particular, starting from flat space one obtains the dilatonic Melvin flux tubes [6].

In the same vein, we have found that new solutions of the $U(1)^4$ theory are generated by the transformations

$$\begin{aligned}
g'_{jk} &= \sqrt{\Lambda}\ g_{jk}, \qquad g'_{\varphi\varphi} = \frac{1}{\sqrt{\Lambda}}g_{\varphi\varphi}, \\
e^{-2\eta'} &= \frac{\lambda_1\lambda_3}{\lambda_2\lambda_4}\ e^{-2\eta}, \qquad e^{-2\sigma'} = \frac{\lambda_1\lambda_4}{\lambda_2\lambda_3}\ e^{-2\sigma}, \\
e^{-2\rho'} &= \frac{\lambda_1\lambda_2}{\lambda_3\lambda_4}\ e^{-2\rho}, \\
A'_{(i)\ \varphi} &= \frac{2}{B_i\lambda_i}\left(1+\frac{1}{2}B_iA_{(i)\ \varphi}\right),
\end{aligned} \tag{3.4}$$

where

$$
\begin{aligned}
\Lambda &= \prod_{i=1}^{4} \lambda_i, \\
\lambda_{1/3} &= \left(1 + \frac{1}{2} B_{1/3} A_{(1/3)\,\varphi}\right)^2 + \frac{1}{4} B_{1/3}^2 g_{\varphi\varphi} e^{\eta \pm (\sigma+\rho)}, \qquad (3.5) \\
\lambda_{2/4} &= \left(1 + \frac{1}{2} B_{2/4} A_{(2/4)\,\varphi}\right)^2 + \frac{1}{4} B_{2/4}^2 g_{\varphi\varphi} e^{-\eta \pm (-\sigma+\rho)}.
\end{aligned}
$$

The proof of invariance of the action (1.2) under these transformations is given in the appendix.

If we apply (3.4) to an asymptotically flat space, then each of the factors λ_i corresponds to a background field that asymptotes to the Melvin solution of $a = \sqrt{3}$ (Kaluza-Klein) theory. The simplest example, in fact, is obtained by applying the transformations to Minkowski space. Then we find

$$
\begin{aligned}
ds^2 &= \sqrt{\lambda_1\lambda_2\lambda_3\lambda_4}(-dt^2 + dz^2 + d\rho^2) + \frac{\rho^2}{\sqrt{\lambda_1\lambda_2\lambda_3\lambda_4}} d\varphi^2, \qquad (3.6) \\
A_{(i)\,\varphi} &= -\frac{B_i \rho^2}{2\lambda_i}, \qquad \lambda_i = 1 + \frac{1}{4}\rho^2 B_i^2, \quad i = 1, \ldots, 4,
\end{aligned}
$$

where the scalars are easily found from the formulas above and the gauge potentials have been shifted by a gauge transformation to make them regular at the axis. In the form given, the fields B_1, B_3 are of magnetic type, whereas B_2, B_4 are electric. The Melvin solutions corresponding to $B_1, B_3 \neq 0$ and $B_2 = B_4 = 0$ were also constructed in [16], where, interestingly, they have been shown to solve exactly (i.e., to all orders in α') the equations of motion of the string. The electric flux tubes B_2, B_4, are formally related to B_1, B_3 by S duality. However, due to the absence of any residual supersymmetry, this cannot be invoked to imply exactness of the full solution.

The factorized form of the solution is immediately apparent. In general, it can be already seen in (3.4). It allows for a composite-field interpretation of the Melvin solutions with dilaton couplings $a = 1, 1/\sqrt{3}, 0$ in terms of 2, 3, 4 'basic' $a = \sqrt{3}$ fields [5]. The picture is that of four parallel and concentrical flux tubes along the z axis carrying independent magnetic or electric flux of each gauge field.

The key for a such a configuration to be possible is that the cancellation of crossed non-linear terms in the Ricci tensor (the latter, of course, not being of the same form as (2.7)) that ocurred for the extremal black holes also happens in this case, due to an algebraic relation between Λ and the scalar fields analogous to (2.8). In this case, however, there is no underlying supersymmetric saturation of Bogomolnyi bounds, since the Melvin backgrounds do not admit Killing spinors.

[5]We thank G. Horowitz for first suggesting to us that Melvin fields of the $U(1)^4$ theory should take a factorized form.

A direct generalization to the $U(1)^n$ theory can be naturally expected. Indeed, we find that the Harrison transformation that introduces magnetic background gauge fields in an axisymmetric solution is

$$
\begin{aligned}
g'_{jk} &= \Lambda^{2/n} g_{jk}, \qquad g'_{\varphi\varphi} = \frac{1}{\Lambda^{2/n}}\, g_{\varphi\varphi}, \\
e^{-\sigma'_i} &= \frac{\lambda_i^2}{\Lambda^{2/n}}\, e^{-\sigma_i}, \\
A'_{(i)\,\varphi} &= \frac{2}{B_i\lambda_i}\left(1+\frac{1}{2}B_i A_{(i)\,\varphi}\right), \\
\Lambda &= \prod_{i=1}^{n} \lambda_i, \\
\lambda_i &= \left(1+\frac{1}{2}B_i A_{(i)\,\varphi}\right)^2 + \frac{1}{4}B_i^2 g_{\varphi\varphi} e^{\sigma_i}.
\end{aligned}
\tag{3.7}
$$

Again, the proof is deferred to the appendix. In the particular case where one takes s of the B_i parameters and gauge fields to be equal and sets the rest to zero, these transformations can be seen to reproduce the dilatonic Harrison transformations (3.3) for the values $a^2 = (n-s)/s$.

The solution describing a static black hole in external fields is obtained by applying these transformations to the black hole solutions of the previous section. In the form given, all the fields are magnetic, whereas the charges of the black holes in (2.11) are electric. Thus, we will first dualize the charges of the black hole solutions so as to make them interact non-trivially with the external fields. The solution thus generated is

$$
\begin{aligned}
ds^2 &= \Lambda^{2/n}\left[-\frac{1}{H}dt^2 + H(dr^2 + r^2 d\theta^2)\right] + \frac{H}{\Lambda^{2/n}} r^2 \sin^2\theta d\varphi^2, \\
A_{(i)\,\varphi} &= \frac{2e^{\sigma_i^0/2}}{B_i\lambda_i}\left(1+\frac{1}{2}q_i B_i \cos\theta\right) + k_i, \\
e^{\sigma_i} &= e^{\sigma_i^0}\,\frac{\Delta_i^n}{H}\frac{\Lambda^{2/n}}{\lambda_i^2}, \\
\lambda_i &= \left(1+\frac{1}{2}q_i B_i\cos\theta\right)^2 + \frac{1}{4}B_i^2 r^2 \Delta_i^n \sin^2\theta,
\end{aligned}
\tag{3.8}
$$

with $H,\ \Delta_i$ as in (2.10). The asymptotic values σ_i^0 for the scalars and the constants k_i fixing the position of Dirac strings have been introduced to later match required values.

The metric (3.8) contains, in general, conical singularities (strings or struts) along the axes $\theta = 0, \pi$. In the case where there is a single field acting, the singularities can not be cancelled by any choice of parameters or period $\Delta\varphi$, the reason being that the external force acting on the black hole cannot be balanced. However, equilibrium can be achieved when different fields act. Indeed, to ensure regularity along the axis we require

$$
\Delta\varphi = 2\pi\Lambda^{2/n}|_{\theta=0} = 2\pi\Lambda^{2/n}|_{\theta=\pi}, \tag{3.9}
$$

which means that

$$
\prod_{i=1}^{n}\left(1+\frac{1}{2}q_i B_i\right) = \prod_{i=1}^{n}\left(1-\frac{1}{2}q_i B_i\right), \tag{3.10}
$$

and for small values of q_iB_i this constraint can be expressed as

$$\sum_i q_i B_i \approx 0, \tag{3.11}$$

i.e., the expected balance of classical forces. We will always impose $\frac{1}{2}|q_iB_i| < 1$; the values $|q_iB_i| = 2$ lead to singularities along any of the axes. Eq. (3.10) requires at least two of the products q_iB_i to be non vanishing.

The parameters q_i and B_i are not the physical charge and field; rather, they approximate them for small values of q_iB_i. The physical charge $\hat{q}_i$ is found, as usual, by integration of the field strengths on a sphere surrounding the black hole, with the result

$$\hat{q}_i = q_i e^{\sigma_i^0/2}\, \frac{\Lambda_0^{2/n}}{1-\frac{1}{4}q_i^2B_i^2}, \tag{3.12}$$

where we defined $\Lambda_0 \equiv \Lambda|_{\theta=0} = \Lambda|_{\theta=\pi}$. On the other hand, we would like to define a physical field strength parameter $\widehat{B}_i$ as the value of the field strength on the axis at infinity. However, rather curiously, the latter quantity takes different values along the axis $\theta = 0$ or $\theta = \pi$. One finds

$$\widehat{B}_i^2|_{\theta=0,\pi} = \frac{1}{2}F_{(i)}^2|_{r\to\infty,\,\theta=0,\pi} = B_i^2\frac{e^{\sigma_i^0}}{(\lambda_i|_{\theta=0,\pi})^3}, \tag{3.13}$$

and, if $q_iB_i \neq 0$, then $\widehat{B}_i|_{\theta=0} \neq \widehat{B}_i|_{\theta=\pi}$. What this means is that the lines of force of each gauge field get distorted due to the interaction with the black hole. The flux of the black hole itself is asymmetrically redistributed in the 'forward-backward' directions. For some of the gauge fields the flux lines concentrate more in the 'forward' direction, for others in the 'backward' direction, but there always remains a residual 'lensing' effect at large distances. Notice, however, that $\prod_i \widehat{B}_i|_{\theta=0} = \prod_i \widehat{B}_i|_{\theta=\pi}$.

In fact, the scalar fields also tend to different values along the axes. As a consequence of this, it is not possible to match the solution to an asymptotic Melvin background simultaneously at both semiaxes $\theta = 0$ and $\theta = \pi$. For definiteness we choose to match at $\theta = 0$, and set

$$e^{\sigma_i^0} = \frac{(\lambda_i|_{\theta=0})^2}{\Lambda_0^{2/n}}. \tag{3.14}$$

Then, at large spatial distances close to the axis $\theta = 0$, the geometry becomes

$$\begin{aligned} ds^2 &\approx \tilde{\Lambda}^{2/n}(-d\bar{t}^2 + d\bar{r}^2 + \bar{r}^2d\theta^2) + \frac{\bar{r}^2\sin^2\theta}{\tilde{\Lambda}^{2/n}}d\bar{\varphi}^2, \\ \tilde{\Lambda} &= \prod_i \tilde{\lambda}_i = \prod_i \left(1 + \frac{1}{4}(\widehat{B}_i^0)^2\bar{r}^2\sin^2\theta\right), \end{aligned} \tag{3.15}$$

with $(\bar{t},\bar{r}) = \Lambda_0^{1/n}(t,r)$, $\bar{\varphi} = \Lambda_0^{-2/n}\varphi$ and $\widehat{B}_i^0 \equiv \widehat{B}_i|_{\theta=0}$. This is precisely a Melvin universe with magnetic field parameter $\widehat{B}_i^0$. However, in the opposite direction we find

$$\widehat{B}_i|_{\theta=\pi} = \widehat{B}_i^0(1+\hat{q}_i\widehat{B}_i^0)^3. \tag{3.16}$$

It is interesting to find that, in terms of the physical parameters, the no-strut condition (3.10) is simply

$$\prod_{i=1}^{n}(1+\hat{q}_i\widehat{B}_i^0)=1. \tag{3.17}$$

One can easily see that the black hole horizon is distorted due to the external fields. The simplest way to see this is by analyzing how the size of the circles $\theta=$ constant changes from the equator to the poles. A more detailed study, which can be easily carried out for, e.g., the case with $n=2$ gauge fields, involves computing the intrinsic scalar curvature of the horizon. The result in the general case is that the curvature is biggest at the poles $\theta=0,\pi$, and decreases monotonically to its smallest value at the equator $\theta=\pi/2$. Furthermore, the curvature of the horizon is always positive. This means that the the horizon is elongated along the axis, and always keeps a convex shape (in particular, it never develops any 'neck' at the equator). However, even if the horizon is distorted, the area

$$\begin{aligned} A_{bh} &= \int d\varphi\, d\theta \sqrt{g_{\theta\theta}g_{\varphi\varphi}}|_{r=0} = 4\pi\Lambda_0^{2/n}\prod_i q_i^{2/n} \\ &= 4\pi\prod_i |\hat{q}_i|^{2/n} \end{aligned} \tag{3.18}$$

(we have used the no-strut condition) is still given by the same function (2.19) of the *physical* charges as in the absence of external fields! The way in which this result appears is rather non trivial, and it would hardly have been expected were it not for the correspondence of the area with a number of microstates.

4 Accelerating composite black holes

The analysis of the previous section has been restricted to static configurations that satisfy the no-strut condition (3.17). In physical terms, the appearance of conical singularities is a rather general feature of configurations where, loosely speaking, there is a mismatch between the acceleration of an object and the forces acting on it. One could say that the conical singularities supply the forces needed to satisfy the equations of motion (operationally, a conical defect δ can be blown up and replaced with a cosmic string vortex with energy density $\mu=\delta/(8\pi)$). We would like to extend our analysis of composite black holes to situations where the black holes accelerate. Configurations of this kind are described in general relativity by the C-metric [17] and Ernst [18] solutions. In fact, they describe the uniformly accelerated motion in opposite directions of *two* oppositely charged black holes. In the C-metric the black holes accelerate without an external force field, and thus the geometry contains strings or struts. A possible way to remove them is by introducing external gauge fields that provide the accelerating force. Technically, this is done by means of a Harrison transformation of the kind discussed above, and the Ernst solution is thus obtained.

The C-metric, and its dilatonic and $U(1)^2$ generalizations given in [20, 21], are of a rather more complicated form than the static, spherically symmetric black hole solutions.

Nevertheless, we will still be able to construct solutions describing extremal composite black holes in accelerated motion. The composites are found to accelerate as a whole, all the constituents remaining bound. The $U(1)^4$ C-metric was already given in [11]. The solution for arbitrary number n of $U(1)$ magnetic charges turns out to be a rather straightforward generalization of it:

$$
\begin{aligned}
ds^2 &= \frac{1}{A^2(x-y)^2}\Big[\mathcal{F}(x)\left(\frac{1-y^2}{\mathcal{F}(y)}dt^2 - \frac{\mathcal{F}(y)}{1-y^2}dy^2\right) \\
&+ \mathcal{F}(y)\left(\frac{\mathcal{F}(x)}{1-x^2}dx^2 + \frac{1-x^2}{\mathcal{F}(x)}d\varphi^2\right)\Big], \\
\mathcal{F}(\xi) &= \prod_{i=1}^{n} f_i(\xi), \qquad f_i(\xi) = (1-q_iA\xi)^{2/n}, \\
A_{(i)\,\varphi} &= q_i x \frac{\sqrt{1-q_i^2A^2}}{f_i(x)^{n/2}}, \\
e^{-\sigma_i} &= \frac{f_i(x)^n\mathcal{F}(y)}{f_i(y)^n\mathcal{F}(x)}.
\end{aligned}
\tag{4.1}
$$

The composite nature of the solution is most clearly visible from the factorized form of the function $\mathcal{F}(y)$. Once more, this is possible due to an algebraic relation similar to (2.13), and is not related to any unbroken supersymmetry, which is absent from this solution. The extremal $U(1)^2$ and dilatonic C-metrics for rational values of a^2 are obtained as particular cases, but to bring them into this form some rewriting of the parameters and coordinates is needed. A very convenient feature of the form we use is that all the roots of the functions involved are easily found exactly.

A brief account of how the C-metrics describe two black holes accelerating apart may be helpful (see [17]). We restrict the parameters to satisfy $|q_iA| < 1\ \forall i = 1,\ldots,n$ [6]. Also, in this section all the q_i will be taken to be non-negative. Then g_{tt}, as a function of y, has three real zeros at $y = -\infty,\ -1,\ 1$ [7].

That the solution contains black holes that can be identified with the static ones can be seen by realizing that the solution (2.4) appears in the limit where $A \to 0$ if we set $y \to -1/(rA)$, $t \to At$. In fact, the zero of g_{tt} at $y = -\infty$ corresponds to the extremal horizon at $r = 0$. It could be brought to a finite coordinate distance by a simple change of coordinates. On the other hand, the zero at $y = -1$ is an acceleration horizon, and the parameter A roughly measures the acceleration of the black holes, and also their separation when they become closest, which is given by $2/A$. The mass of the black holes can only be defined in an approximate way, for small accelerations, as $M \approx \frac{1}{n}\sum_i q_i$. Also, the coordinate patch chosen covers only the region of space time containing one of the black holes. The

[6]The claim in [11] that an isometry relates the solution with $|q_iA| < 1$ to that with $|q_iA| > 1$ was not correct.

[7]If either none or half of the charges vanish, there is an additional real zero at $+\infty$, which can be identified with the one at $-\infty$ and which, in any case, will be of no relevance for us.

solution can be continued to yield the remaining Rindler wedge containing an oppositely charged black hole.

In the static limit x plays the role of $\cos\theta$. In general, when x is restricted to $-1 \leq x \leq 1$ and φ is periodically identified, the (x, φ) sector has topology S^2. The roots $x = -1, x = 1$ correspond to the poles of the sphere, and define the axes pointing to infinity and to the other black hole, respectively. Asymptotic infinity is at the point $x = y$ where the conformal factor in front of the metric diverges; in particular, $x = y = -1$ is spacelike infinity. Hence y is restricted to $-\infty < y < x$. With coordinates and parameters within this range, the metric has the appropriate Lorentzian signature.

The solution asymptotes to flat space, with conical singularities along the axes $x = \pm 1$ that, for non-negative values of q_i, cannot be simultaneously cancelled everywhere. As explained, an external field can provide the additional parameter needed to construct regular solutions. Thus, we perform a $U(1)^n$-Harrison transformation (3.7) on (4.1) to get the $U(1)^n$ Ernst solution,

$$\begin{aligned}
ds^2 &= \frac{\Lambda^{2/n}}{A^2(x-y)^2}\left[\mathcal{F}(x)\left(\frac{1-y^2}{\mathcal{F}(y)}dt^2 - \frac{\mathcal{F}(y)}{1-y^2}dy^2\right) + \mathcal{F}(y)\frac{\mathcal{F}(x)}{1-x^2}dx^2\right] \\
&+ \frac{(1-x^2)\mathcal{F}(y)}{A^2(x-y)^2\Lambda^{2/n}\mathcal{F}(x)}d\varphi^2, \\
A_{(i)\,\varphi} &= \frac{2e^{\sigma_i^0/2}}{B_i\lambda_i}\left(1 + \frac{1}{2}q_i B_i x\frac{\sqrt{1-q_i^2A^2}}{f_i(x)^{n/2}}\right) + k_i, \\
e^{-\sigma_i} &= e^{-\sigma_i^0}\frac{f_i(x)^n\mathcal{F}(y)}{f_i(y)^n\mathcal{F}(x)}\frac{\lambda_i^2}{\Lambda^{2/n}}, \\
\lambda_i &= \left(1 + \frac{1}{2}q_i B_i x\frac{\sqrt{1-q_i^2A^2}}{f_i(x)^{n/2}}\right)^2 + \frac{1}{4}B_i^2\frac{1-x^2}{A^2(x-y)^2}\frac{f_i(y)^n}{f_i(x)^n},
\end{aligned} \tag{4.2}$$

with $\mathcal{F}(\xi)$, $f_i(\xi)$ as above. Non vanishing σ_i^0's have been reintroduced and will be determined later.

Conical singularities in the spheres (x, φ) can now be removed everywhere if

$$\Delta\varphi = 2\pi\Lambda(1)^{2/n}\mathcal{F}(1) = 2\pi\Lambda(-1)^{2/n}\mathcal{F}(-1) \tag{4.3}$$

where $\Lambda(\pm 1) \equiv \Lambda(x = \pm 1)$. This equation admits non-trivial solutions if at least one of the products $q_i B_i$ is non-zero. For small values of the q_i it yields, in terms of the 'canonical' fields and charges (1.5), $MA \approx \sum_i \mathcal{Q}_i\mathcal{B}_i$, i.e., Newton's law.

The solution can be seen to asymptote to a Melvin field at spatial infinity $x, y \to -1$. To this effect, change to coordinates $(\bar{t}, \bar{y}, \bar{x}, \bar{\varphi})$ using

$$x + 1 = -2L^{4/n}\mathcal{F}(-1)^2\frac{1-\bar{x}^2}{(\bar{x}+\bar{y})^2},$$

$$
\begin{aligned}
y+1 &= -2L^{4/n}\mathcal{F}(-1)^2\frac{1-\bar{y}^2}{(\bar{x}+\bar{y})^2}, \\
t &= \mathcal{F}(-1)\,\bar{t}, \quad \varphi = \mathcal{F}(-1)L^{4/n}\,\bar{\varphi},
\end{aligned}
\tag{4.4}
$$

where $L^2 \equiv \Lambda(x=-1)$. For $x, y \to -1$ the $U(1)^n$-Ernst metric then goes to

$$
\begin{aligned}
ds^2 \;&\approx\; \frac{\tilde{\Lambda}^{2/n}}{A^2(\bar{x}-\bar{y})^2}\left[(1-\bar{y}^2)d\bar{t}^2 - \frac{d\bar{y}^2}{1-\bar{y}^2} + \frac{d\bar{x}^2}{1-\bar{x}^2}\right] \\
&+ \frac{1-\bar{x}^2}{A^2(\bar{x}-\bar{y})^2\tilde{\Lambda}^{2/n}}d\bar{\varphi}^2,
\end{aligned}
\tag{4.5}
$$

with

$$
\begin{aligned}
\widehat{B}_i \;&=\; \frac{B_i}{\ell_i L^{2/n}\mathcal{F}(-1)}, \qquad \ell_i^2 = \lambda_i(x=-1) \\
\tilde{\Lambda} \;&=\; \prod_{i=1}^{n}\left(1+\frac{1}{4}\widehat{B}_i^2\frac{1-\bar{x}^2}{A^2(\bar{x}-\bar{y})^2}\right).
\end{aligned}
\tag{4.6}
$$

This is a Melvin background with magnetic field parameter $\widehat{B}_i$, though written in somewhat unusual accelerated coordinates [22]. The scalar and gauge fields can be matched if we set

$$
e^{\sigma_i^0} = \frac{\ell_i^4}{L^{4/n}}. \tag{4.7}
$$

With this value, the magnetic field on the axis $x=-1$ at large spatial distances, $y \to -1$,

$$
\frac{1}{2}F_{(i)}^2|_{x=-1} \to B_i^2\frac{e^{\sigma_i^0}}{\ell_i^6\mathcal{F}(-1)^2}, \tag{4.8}
$$

is consistent with the physical field parameter in (4.6).

As of the physical charges of the black holes in the $U(1)^n$-Ernst metric, they are readily computed as

$$
\hat{q}_i = q_i\,\frac{\mathcal{F}(-1)L^{2/n}\ell_i}{\sqrt{1-q_i^2A^2}\sqrt{\lambda_i(x=1)}}. \tag{4.9}
$$

We have already found that external fields do not change the area (i.e., the entropy) of the extreme black holes. Does the acceleration affect it? The answer,

$$
\begin{aligned}
A_{bh} \;&=\; \int d\varphi\, dx\sqrt{g_{xx}g_{\varphi\varphi}}|_{y=-\infty} = 4\pi\Lambda_0^{2/n}\mathcal{F}(1)\prod_i q_i^{2/n} \\
&=\; 4\pi\prod_i|\hat{q}_i|^{2/n},
\end{aligned}
\tag{4.10}
$$

once again, shows that the dependence on the physical charges remains unaltered.

The shape of the black holes can be studied as we did in the static case, but the formulas get rather complicated. The result is that, in the general situaton where there are at least two non-equal gauge fields, the black holes are again elongated along the axis (this was also

noticed in [21]). Curiously, when there is effectively only one gauge field excited, i.e., when all the non-vanishing gauge fields are equal, the solution near the horizon approaches exactly the spherically symmetric static black hole solution [22]. At present it is not clear to us why this should be so.

The Lorentzian solutions just described can be easily continued to imaginary time $\tau = it$. When y has range $-\infty < y \leq -1$ and τ is periodically identified to satisfy regularity at the acceleration horizon, i.e.,

$$\Delta\tau = 2\pi\mathcal{F}(-1), \tag{4.11}$$

we get an exact Euclidean instanton with topology $S^2 \times \mathbf{R}^2 - \{pt\}$ mediating the pair production of these extreme black holes. For the particular case of the $U(1)^4$ theory, this includes, e.g., the pair creation of Kaluza-Klein monopoles [22] and H-monopoles.

The pair creation rate is calculated, in the leading approximation, as $\Gamma \sim e^{-I}$, with I the classical Euclidean action of the instanton. This quantity is given by the usual boundary terms at infinity, after having matched the boundary geometry and fields to those of the Melvin background [23]. The Hamiltonian vanishes, and the action is fully given by the difference between the areas of the acceleration horizons in the $U(1)^n$ Ernst metric and in the reference Melvin space. The final exact answer takes the form

$$I = -\frac{1}{4}\Delta A_{acc} = \frac{\pi L^{4/n}\mathcal{F}(-1)^2}{nA}\sum_{i=1}^{n}\frac{q_i}{1+q_iA}. \tag{4.12}$$

This result can be seen to agree with those previously found for the particular cases considered in [22, 21]. For small charges, and using the no strut condition, one finds

$$I \approx \frac{\pi M^2}{\sum_{i=1}^{n}\mathcal{Q}_i\mathcal{B}_i}, \tag{4.13}$$

i.e., the generalized Schwinger value $I \approx \pi M^2/F$, with F the driving force.

We should mention that, as in [23], we have not included the contribution from the extreme black hole area in the calculation of the instanton action above. This term, if added in the cases where the black hole horizon is regular, would yield an enhancement of pair creation rates due to macroscopic indistinguishability of the internal states of the black holes created. Since the extremal black holes, at least the stringy ones, appear to have non-zero entropy, the term $-A_{bh}/4$ should presumably be added to (4.12) to find the pair creation rate.

Notice that the sign of the instanton action (4.12) is controlled by the term $\sum_i q_i/(1+q_iA)$ (recall that we only allow $|q_iA| < 1$ and therefore $\mathcal{F}(\pm 1) > 0$). This implies that I is always positive for non-negative q_i. In next section we will discuss a different situation.

5 Massless holes

Zero-mass extreme black holes have recently attracted interest. We would like, however, to distinguish between two different objects, both referred to as massless black holes, that have

been described in the literature and whose relationship is not clear. Strominger showed in [24] that certain singularities (conifold points) of the moduli space of string vacua could be understood as the effect of extremal black holes becoming massless. This is very similar to the mechanism by which phase transitions in $N = 2$ Yang-Mills are triggered by condensation of massless BPS monopoles. It must be stressed, though, that since the Lagrangian is singular at the conifold, these massless black holes do not admit a semiclassical description.

The other class of massless black holes corresponds to nakedly singular solutions of a regular classical Lagrangian [25, 26, 27]. It is not clear whether these solutions can be consistently identified with the massless solitons of [24]. They preserve a fraction of the supersymmetries of the theory, but, on account of their naked singularities, they also present a number of bizarre properties. These holes, though massless, are found to follow timelike trajectories, and in fact they can be at rest. Since their effect on test particles is repulsive [26] we will avoid to refer to them as 'black'.

Single-scalar-Maxwell theories do not have non-trivial extreme massless solutions: the extreme black holes in these theories are essentially determined by only one parameter, and the mass can not be set to zero without also setting to zero the other charges. As was argued in [12], massless holes can be easily found in theories admitting composite black holes. All one has to do is to allow for negative 'constituent masses', something not evidently inconsistent as long as the total mass of the composite is non-negative. Thus, for example, in the $U(1)^4$ theory we can choose $q_1 = -q_3 > 0$ and $q_2 = q_4 = 0$ to get a solution with zero ADM mass. In this case, a naked curvature singularity appears at $r = -q_3$, but the Bogomolnyi bound can still be non-trivially saturated. The same thing applies to the $U(1)^n$ theories, which admit a large variety of such solutions for any $n \geq 2$, with a naked singularity at $r = \max_i\{-q_i\}$.

We have not found any surprises in the analysis of static massless solutions subject to external fields. However, the accelerating solutions do indeed present a striking behavior. For the $U(1)^4$ theory they were studied in [11]. Here we will extend the results to $U(1)^n$ theories.

Consider the $U(1)^n$ C-metric (4.1). No external field acts upon the black hole, and the solution asymptotes to Minkowski space. The no-strut condition reads

$$\Delta\varphi = 2\pi\mathcal{F}(1) = 2\pi\mathcal{F}(-1). \tag{5.1}$$

This equation can be non-trivially solved only if negative values of the q_i are allowed. In fact, the solution for arbitrary values of A requires

$$\begin{aligned} \sum_{i=1}^{n} q_i &= 0, \qquad (5.2)\\ \sum_{i<j<k} q_i q_j q_k &= 0,\\ &\vdots\\ \sum_{i_1<i_2<\ldots<i_r} q_{i_1}\ldots q_{i_r} &= 0 \quad \text{for all odd } r \leq n. \end{aligned}$$

In particular, the first equation (5.2) implies that the hole has zero mass [8]. Hence, objects satisfying these conditions accelerate freely and uniformly in the otherwise empty Minkowski space. Notice that it is not clear at all how to change the state of motion of a particle that is following a timelike trajectory but nevertheless has zero mass. This is another indication that, although they are classical solutions, the massless holes exhibit a behavior markedly different from that of any classical particle.

Since some of the q_i are negative, these solutions have a naked singularity at $y = \max_i\{(q_i A)^{-1}\}$, equivalent to the singularity of the massless static solution at $r = \max_i\{-q_i\}$. If the latter is to be regarded as acceptable, then we would expect the Euclidean solution obtained from the C-metric to be a valid instanton mediating the pair creation of these massless objects. This would describe a putative decay of Minkowski space.

The Euclidean action of the instanton can be readily obtained from (4.12) by setting the fields B_i to zero and enforcing the conditions (5.2). The result, that generalizes the one in [11], can be given in a simple, exact form as

$$I = -\frac{\pi}{n}\sum_{i=1}^{n}\hat{q}_i^2, \tag{5.3}$$

i.e., it is essentially given by the mean square physical charge. *This action is negative.* Such result could not have been possible had not we allowed for naked singularities. It seems to imply the undesired conclusion that Minkowski space would be wildly unstable against pair creation of classical massless particles. Apparently, then, either an argument ruling out the instanton, but not the solutions with static naked singularities, is found, or otherwise the massless solutions should be deemed as unphysical[9].

Similarly, a Melvin background could also decay by forming pairs of massless holes. The action, which can also be read from (4.12), is again given (but now only to leading order for small charges) by the same negative value (5.3). This decay mode, then, would dominate by far over the creation of massive black holes.

We conclude this section by stressing that the purely quantum massless black holes of [24] are not affected in principle by this result. We can not use a classical action to construct a decay instanton, since the Lagrangian becomes singular when the massless black holes appear. The same thing could be said of massless non-abelian monopoles if we tried to analyze their pair creation along the lines of [28].

6 Non-extremal static and accelerated black holes

Thus far we have been concentrating only on the solutions describing extreme black holes. These are the ones that allow for a proper composite interpretation, and their entropy can be

[8] Or, rather, since the mass is not defined in any precise way for the accelerating object, we should say that there is a one-to-one correspondence with the static massless solution in the limit $A \to 0$.

[9] Some possible objections to this line of reason were considered in [11].

defined solely in terms of conserved charges. Nevertheless, it is possible to construct the non-extreme versions of these black holes for the general $U(1)^n$ theory in a rather straightforward way. Remarkably enough, the corresponding non-extremal C-metrics and Ernst solutions can also be easily found.

Let us start by writing the non-extreme dilaton black hole solution of (1.1) in the following convenient way

$$\begin{aligned} ds^2 &= -\frac{1-r_0/r}{(1+\beta/r)^{2/(1+a^2)}}dt^2 + \frac{(1+\beta/r)^{2/(1+a^2)}}{1-r_0/r}dr^2 + r^2\left(1+\frac{\beta}{r}\right)^{2/(1+a^2)}d\Omega^2, \quad (6.1)\\ e^{-2a\phi} &= \left(1+\frac{\beta}{r}\right)^{2a^2/(1+a^2)}, \qquad A_t = -\frac{Q}{\beta}\left(1+\frac{\beta}{r}\right)^{-1}, \end{aligned}$$

where we have set $\phi_0 = 0$, and the electric charge is

$$Q = \beta\sqrt{\frac{1+r_0/\beta}{1+a^2}}. \tag{6.2}$$

The outer horizon is at $r = r_0$, and the inner horizon at $r = 0$. The parameter r_0 is equal to zero at extremality.

We observe that this solution can be obtained from the extreme one by essentially introducing the factor $1 - r_0/r$ in g_{tt} and g_{rr}. Is this feature generalizable to the $U(1)^n$ theory? We find that the answer is yes. The simple modification

$$\begin{aligned} ds^2 &= -\frac{1-r_0/r}{H}dt^2 + \frac{H}{1-r_0/r}dr^2 + r^2 H d\Omega^2, \quad (6.3)\\ A_{(i)\,t} &= \frac{\sqrt{1+r_0/q_i}}{\Delta_i^{n/2}}, \end{aligned}$$

with H, Δ_i and the scalar fields as in the extreme solution (2.10), (2.11), turns out to be the correct solution. The electric charges are in this case

$$\hat{q}_i = q_i\sqrt{1+\frac{r_0}{q_i}}, \tag{6.4}$$

and the mass

$$M = \frac{r_0}{2} + \frac{1}{n}\sum_{i=1}^{n} q_i = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{q}_i^2 + \frac{r_0^2}{4}\right)^{1/2} \tag{6.5}$$

does not saturate any longer the extremal bound. For four $U(1)$ charges this solution was previously found in [29]. By taking s equal nonvanishing charges, with the remaining ones being zero, we reproduce, with the same identifications as in (2.17), the non-extreme dilaton black holes (6.1).

The horizon area of these black holes,

$$A_{bh} = 4\pi\prod_{i=1}^{n}(q_i + r_0)^{2/n} = 4\pi\prod_{i=1}^{n}\left[\left(\hat{q}_i^2 + \frac{r_0^2}{4}\right)^{1/2} + \frac{r_0}{2}\right]^{2/n}, \tag{6.6}$$

depends on r_0 and cannot be expressed in terms of conserved charges only.

With little extra effort we can find non-extreme C-metrics. Once again, the simplest possibility turns out to yield the correct answer,

$$\begin{aligned} ds^2 &= \frac{1}{A^2(x-y)^2}\Big[\mathcal{F}(x)\left(\frac{(1-y^2)(1+r_0Ay)}{\mathcal{F}(y)}dt^2 - \frac{\mathcal{F}(y)}{(1-y^2)(1+r_0Ay)}dy^2\right) \\ &+ \mathcal{F}(y)\left(\frac{\mathcal{F}(x)}{(1-x^2)(1+r_0Ax)}dx^2 + \frac{(1-x^2)(1+r_0Ax)}{\mathcal{F}(x)}d\varphi^2\right)\Big], \qquad (6.7) \\ A_{(i)\,\varphi} &= q_i x\frac{\sqrt{(1+r_0/q_i)(1-q_i^2A^2)}}{f_i(x)^{n/2}}. \end{aligned}$$

$\mathcal{F}$, f_i and the scalars are as in (4.1). In the form given, the black holes have magnetic charge. Again, to bring this solution into the form of the particular cases considered in [20, 21], the coordinates and parameters have to be somewhat transformed.

For non-negative q_i's, we now find four real zeroes of g_{tt} at $y = -\infty,\ -1/(r_0A),\ -1,\ +1$. With $0 \le r_0A < 1$ and $-\infty < y < x$, $-1 \le x \le 1$, the solution describes a non-extreme black hole and its antiparticle accelerating in opposite directions; $y = -\infty, -1/(r_0A), -1$ are, respectively, the inner horizon and outer horizon of the black hole, and the acceleration horizon.

It is now easy to perform a $U(1)^n$ Harrison transformation on any of these solutions, and thus introduce background Melvin fields. In particular, in this way we obtain non-extreme $U(1)^n$ Ernst solutions, that can be continued to imaginary time so as to find instantons for non-extreme black hole pair creation. Euclidean regularity requires matching the temperature of black hole and acceleration horizons. Since the analysis is rather straightforward, we will not give further details, and shall only quote the value of the instanton action mediating non-extreme black hole pair production. After imposing the instanton regularity conditions, and adding the contribution from the black hole area, the exact result is found to be

$$I = -\frac{1}{4}(\Delta A_{acc} + A_{bh}) = \frac{\pi L^{4/n}\mathcal{F}(-1)^2}{nA(1-r_0A)}\sum_{i=1}^{n}\frac{q_i}{1+q_iA}, \qquad (6.8)$$

with

$$L = \prod_{i=1}^{n}\left(1 - \frac{1}{2}q_iB_i\sqrt{\frac{(1+r_0/q_i)(1-q_iA)}{1+q_iA}}\right). \qquad (6.9)$$

Again, this can be readily seen to agree, for small $r_0,\ q_i$, with Schwinger's result (4.13).

7 Concluding remarks

We would like to highlight two results from this analysis of composite black holes in external fields. First, the invariance of the extremal black hole entropy as expressed in terms of conserved charges alone, and, second, the fact that the compositeness property is not

restricted to solutions preserving some supersymmetry, but also to backgrounds that do not admit Killing spinors, like Melvin flux tubes or C-metrics. The compositeness seems to be better related to algebraic relations of the type of (2.13), which only for some classes of solutions (static extreme black holes) coincide with saturated Bogomolnyi bounds. This algebraic property, in particular, makes it possible to construct solutions containing an arbitrary number of parameters even for cases as complex as the Ernst solutions.

By introducing an arbitrary number of gauge fields we have been able to extend the composite interpretation to all the dilatonic black holes with rational value of a^2. One may wonder whether this cannot be extended to all real values of a. Formally, this can be done by allowing for an *infinite* number of gauge fields. Though the consistency of such a theory might be problematic, it is amusing to see how this would work. Consider, then, replacing the discrete index i in the action (1.3) with a continuous parameter ξ with range $0 \le \xi \le 1$, and let the fields $\sigma(\xi)$, $F_{\mu\nu}(\xi)$ depend on it. The new action is

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big\{R - \frac{1}{2}\int_0^1 d\xi_1 \int_{\xi_1}^1 d\xi_2 \left[\partial\sigma(\xi_1) - \partial\sigma(\xi_2)\right]^2 - \int_0^1 d\xi\; e^{-\sigma(\xi)}F(\xi)^2\Big\}, \quad (7.1)$$

with, now, $\int_0^1 d\xi\;\sigma(\xi) = 0$. It is curious that, if we turn off the gravity and scalar interactions, the Maxwell field action can be seen as defined on a five dimensional space $M_4 \times I$, with I the unit interval, and $F_{\mu\xi} = 0$. This interpretation, though, does not seem to be generalizable to the whole action (7.1).

Extreme black hole solutions of this theory are given in terms of a function $q(\xi)$ which we take to be non-negative. The metric is of the same form (2.6), but now with

$$\log H = 2\int_0^1 d\xi\;\log\left(1 + \frac{q(\xi)}{r}\right) \quad (7.2)$$

and

$$e^{-\sigma(\xi)} = \frac{\left(1 + q(\xi)/r\right)^2}{H}, \qquad A_t(\xi) = \left(1 + \frac{q(\xi)}{r}\right)^{-1}. \quad (7.3)$$

The mass is $M = \int d\xi\; q(\xi)$, and the horizon area

$$A_{bh} = 4\pi \exp\left(2\int_0^1 d\xi\;\log q(\xi)\right). \quad (7.4)$$

To reproduce the dilaton black holes with generic coupling a, take $q(\xi)$ to be a step function,

$$\begin{aligned} q(\xi) &= q \quad \text{for } 0 \le x < \frac{1}{1+a^2}, \\ &= 0 \quad \text{for } \frac{1}{1+a^2} < x \le 1. \end{aligned}$$

The formal generalization of all other results in the paper is straightforward.

A different 'origin' for extreme dilatonic black holes with $a = \sqrt{p/(p+2)}$ has been given in [30], where, for p odd, they are found to arise in a non-singular way from non-dilatonic p

branes in $(4+p)$ dimensions. It would certainly be interesting to find the connection between their approach and the one in this paper. In a similar vein, we believe that generalizations of the $U(1)^n$ action (1.3) should also exist at least for five dimensions. Although C-metrics are known essentially only for four dimensional black holes, Melvin flux tubes are more amenable to generalizations to higher dimensions. Thus, it appears that the analysis carried out in Sec. 3 could be extended to higher dimensional black holes (specifically, five dimensional), and extended objects like p-branes (for related extensions, see [31]). Results similar to those presented in this paper are expected to hold.

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A $U(1)^n$ Harrison transformations

In this appendix we outline the proof that the generalized Harrison transformations (3.4) and (3.7) leave the actions (1.2) and (1.3), respectively, invariant.

In fact, the proof proceeds by direct check. However, the algebra can be simplified by rewriting the action in an appropriate manner. Since the truncated heterotic $U(1)^4$ action can be mapped into the $U(1)^n$ action by identifying fields as in (2.24), we will only present the result for the latter case.

Given an axisymmetric solution of the form (3.1), we denote

$$g_{\varphi\varphi} = V, \qquad g_{jk} = {}^3g_{jk}, \tag{A.1}$$

and then integrate the cyclic coordinate φ in the action. We obtain the following effective three-dimensional action

$$I = \frac{\Delta\varphi}{16\pi G}\int d^3x\sqrt{-{}^3g}V^{1/2}\Big\{{}^3R - \frac{1}{2n^2}\sum_{i<j}(\partial\sigma_i - \partial\sigma_j)^2 - \frac{2}{nV}\sum_{i=1}^{n} e^{-\sigma_i}(\partial A_{(i)\varphi})^2\Big\}. \tag{A.2}$$

The transformations (3.7) act conformally on the metric ${}^3g_{jk}$. For the $U(1)^4$ action (1.2) one might also consider writing things in terms of the conformally related string metric $d\bar{s}^2 = e^{\eta}ds^2$, but due to the presence of other scalar fields this turns out to be not very advantageous. Lengthy but straightforward algebra shows now that the action (A.2) is left invariant under these transformations.

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